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MHD Boundary Layer Flow Due to an Exponentially Stretching Sheet with Radiation Effect

(Aliran Lapisan Sempadan MHD Berpunca daripada Regangan Helaian Secara Eksponen dengan Kesan Sinaran)

ANUAR ISHAK

ABSTRACT

The effect of radiation on magnetohydrodynamic (MHD) boundary layer flow of a viscous fluid over an exponentially stretching sheet was studied. The governing system of partial differential equations was transformed into ordinary differential equations before being solved numerically by an implicit finite-difference method. The effects of the governing parameters on the flow field and heat transfer characteristics were obtained and discussed. It was found that the local heat transfer rate at the surface decreases with increasing values of the magnetic and radiation parameters.

Keywords: Boundary-layer; heat transfer; MHD; radiation; stretching sheet

ABSTRAK

Kesan sinaran terhadap aliran lapisan sempadan magnetohidrodinamik (MHD) bagi bendalir likat di atas helaian meregang secara eksponen dikaji. Sistem persamaan menakluk dalam bentuk persamaan pembezaan separa dijelmakan kepada persamaan pembezaan biasa sebelum diselesaikan secara berangka menggunakan kaedah beza-terhingga tersirat. Kesan parameter-parameter menakluk terhadap ciri-ciri aliran dan pemindahan haba diperoleh dan dibincangkan. Didapati bahawa kadar pemindahan haba setempat pada permukaan berkurang dengan pertambahan nilai parameter magnet dan parameter sinaran.

Kata kunci: Lapisan sempadan; MHD; pemindahan haba; regangan helaian; sinaran

INTRODUCTION

The boundary layer flow on a continuous stretching sheet has attracted considerable attention during the last few decades due to its numerous applications in industrial manufacturing processes such as hot rolling, wire drawing, glass-fiber and paper production, drawing of plastic films, metal and polymer extrusion and metal spinning. Both the kinematics of stretching and the simultaneous heating or cooling during such processes has a decisive influence on the quality of the final products (Magyari & Keller 1999).

Crane (1970) was the first to consider the boundary layer flow caused by a stretching sheet which moves with a velocity varying linearly with the distance from a fixed point. The heat transfer aspect of this problem was investigated by Carragher and Crane (1982), under the conditions when the temperature difference between the surface and the ambient fluid is proportional to a power of the distance from a fixed point. Magyari and Keller (1999) investigated the steady boundary layers on an exponentially stretching continuous surface with an exponential temperature distribution. Partha et al. (2005) studied the effect of viscous dissipation on the mixed convection heat transfer from an exponentially stretching surface. Recently, Sajid and Hayat (2008) extended this problem by investigating the radiation effects on the flow over an exponentially stretching sheet, and solved the problem analytically using the homotopy analysis method. The numerical solution for the same problem was then given by Bidin and Nazar (2009).

The study of magnetohydrodynamic has important applications, and may be used to deal with problems such as cooling of nuclear reactors by liquid sodium and induction flow meter, which depends on the potential difference in the fluid in the direction perpendicular to the motion and to the magnetic field (Ganesan & Palani 2004). At high operating temperature, radiation effect can be quite significant. Many processes in engineering areas occur at high temperatures and knowledge of radiation heat transfer becomes very important for the design of pertinent equipment (Seddeek 2002).

Motivated by the above investigations, the present paper studies the problem of MHD boundary layer flow over an exponentially stretching sheet in the presence of radiation, which has not been considered before.

PROBLEM FORMULATION

Consider a steady two-dimensional flow of an incompressible viscous and electrically conducting fluid caused by a stretching sheet, which is placed in a quiescent ambient fluid of uniform temperature T_∞, as shown in Figure 1. We consider that a variable magnetic field *B(x)* is applied normal to the sheet and that the induced magnetic field is neglected, which is justified for MHD flow at small magnetic Reynolds number. Under the usual boundary layer approximations, the flow and heat transfer with the radiation effects are governed by the following equations:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\nu\frac{\partial^2 u}{\partial y^2}-\frac{\sigma B^2}{\rho}u, \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{k}{\rho c_p}\frac{\partial^2 T}{\partial y^2}-\frac{1}{\rho c_p}\frac{\partial q_r}{\partial y}, \tag{3}$$

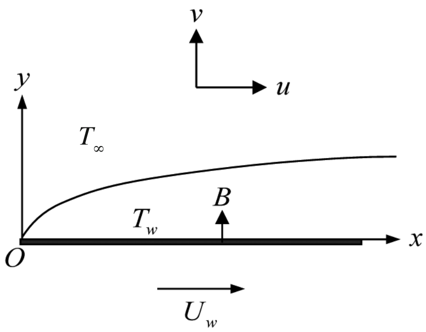

FIGURE 1. Physical model and coordinate system

where u and v are the velocities in the x- and y-directions, respectively, ρ is the fluid density, ν the kinematic viscosity, k the thermal conductivity, c_p the specific heat, T the fluid temperature in the boundary layer and q_r is the radiative heat flux. The boundary conditions are given by:

$$\begin{aligned} &u=U_w=U_0e^{x/L},\ v=0,\\ &T=T_w=T_\infty+T_0e^{x/(2L)}\quad \text{at } y=0,\\ &u\to 0,\ T\to T_\infty \quad \text{as } y\to\infty, \end{aligned} \tag{4}$$

where U_0 is the reference velocity, T_0 the reference temperature and L is the reference length.

Most of the effort in understanding fluid radiation is devoted to the derivation of reasonable simplifications (Aboeldahab & El Gendy 2002). One of these simplifications was made by Cogley et al. (1968) who assumed that the fluid is in the optically thin limit and, accordingly, the fluid does not absorb its own radiation but it only absorbs radiation emitted by the boundaries. For an optically thick gas, the gas self-absorption rises and the situation become difficult. However, the problem can be simplified by using the Rosseland approximation (Rosseland 1936; Siegel & Howell 1992; Sparrow & Cess 1978) which simplifies the radiative heat flux as:

$$q_r=-\frac{4\sigma^*}{3k^*}\frac{\partial T^4}{\partial y}, \tag{5}$$

where σ^* and k^* are the Stefan-Boltzmann constant and the mean absorption coefficient, respectively. This approximation is valid at points optically far from the boundary surface, and, it is good only for intensive absorption, which is for an optically thick boundary layer (Bataller 2008; Siegel & Howell 1992; Sparrow & Cess 1978). It is assumed that the temperature differences within the flow such that the term T^4 may be expressed as a linear function of temperature. Hence, expanding T^4 in a Taylor series about T_∞ and neglecting higher-order terms gives:

$$T^4\approx 4T_\infty^3T-3T_\infty^4. \tag{6}$$

Using Eqs. (5) and (6), Eq. (3) reduces to:

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{k}{\rho c_p}\frac{\partial^2 T}{\partial y^2}+\frac{16\sigma^*T_\infty^3}{3\rho c_p k^*}\frac{\partial^2 T}{\partial y^2}. \tag{7}$$

To obtain similarity solutions, it is assumed that the magnetic field *B(x)* is of the form:

$$B=B_0e^{x/(2L)}, \tag{8}$$

where B_0 is the constant magnetic field.

The continuity equation (1) is satisfied by introducing a stream function ψ such that:

$$u=\frac{\partial\psi}{\partial y}\quad\text{and}\quad v=-\frac{\partial\psi}{\partial x}. \tag{9}$$

The momentum and energy equations can be transformed into the corresponding ordinary differential equations by the following transformation (Sajid & Hayat 2008):

$$\begin{aligned} &u=U_0e^{x/L}f'(\eta),\quad v=-\left(\frac{\nu U_0}{2L}\right)^{1/2}e^{\frac{x}{2L}}\left(f(\eta)+\eta f'(\eta)\right),\\ &T=T_\infty+T_0e^{x/(2L)}\theta(\eta),\quad \eta=\left(\frac{U_0}{2\nu L}\right)^{1/2}e^{\frac{x}{2L}}y, \end{aligned} \tag{10}$$

where η is the similarity variable, $f(\eta)$ is the dimensionless stream function, $\theta(\eta)$ is the dimensionless temperature and primes denote differentiation with respect to η. The transformed ordinary differential equations are:

$$f'''+ff''-2f'^2-Mf'=0, \tag{11}$$

$$\left(1+\frac{4}{3}K\right)\theta''+\Pr\left(f\theta'-f'\theta\right)=0, \tag{12}$$

in which $M=\frac{2\sigma B_0^2L}{\rho U_0}$ is the magnetic parameter, $K=\frac{4\sigma^*T_\infty^3}{k^*k}$ the radiation parameter and $\Pr=\frac{\rho\nu c_p}{k}$ is the Prandtl number. The transformed boundary conditions are:

$$\begin{aligned} &f(0)=0,\ f'(0)=1,\ \theta(0)=1,\\ &f'(\eta)\to 0,\ \theta(\eta)\to 0\quad\text{as}\quad \eta\to\infty. \end{aligned} \tag{13}$$

The main physical quantities of interest are the skin friction coefficient $f''(0)$ and the local Nusselt number $-\theta'(0)$, which represent the wall shear stress and the heat transfer rate at the surface, respectively. Our task is to investigate how the values of $f''(0)$ and $-\theta'(0)$ vary with the radiation parameter K, magnetic parameter M and Prandtl number Pr.

RESULTS AND DISCUSSION

The system of ordinary differential equations (11) – (13) has been solved numerically using the Keller box method described in the book by Cebeci and Bradshaw (1988). This method has been successfully used by the present author to solve various boundary layer problems along with the concept of similarity solution (see Ishak (2009a,b) and Ishak et al. (2008a,b, 2009a,b)). Comparison with the existing results from the literature shows a favourable agreement, as presented in Table 1.

The velocity profiles for different values of the magnetic parameter M presented in Figure 2 show that the rate of transport is considerably reduced with the increase of M. It clearly indicates that the transverse magnetic field opposes the transport phenomena. This is because the variation of M leads to the variation of the Lorentz force due to the magnetic field, and the Lorentz force produces more resistance to the transport phenomena. We note that the Prandtl number Pr and the radiation parameter K have no influence on the flow field, which is clear from Equation (11). The velocity gradient at the surface $f''(0)$ which represents the surface shear stress increases with increasing M. Thus, the magnetic parameter M acts as a controlling parameter to control the surface shear stress.

The temperature profiles for different values of M, K and Pr with other parameters are fixed to unity are presented in Figures 3, 4 and 5, respectively. Figures 2 to 5 show that the far field boundary conditions are satisfied asymptotically, thus supporting the accuracy of the numerical results obtained. It is evident from Figures 3 to 5 that the thermal boundary layer thickness increases as M and K increase but opposite trends are observed for increasing values of Pr. This results in decreasing manner of the local Nusselt number $-\theta'(0)$, which represents the heat transfer rate at the surface, with increasing M and K but opposite trends are observed for increasing values of Pr. This is because, when Pr increases, the thermal diffusivity decreases and thus the heat is diffused away from the heated surface more slowly and in consequence increase the temperature gradient at the surface.

TABLE 1. Values of $\theta'(0)$ for different values of K, M and Pr

K	M	Pr	Magyari and Keller (1999)	El-Aziz (2009)	Bidin and Nazar (2009)	Present results
0	0	1	-0.954782	-0.954785	-0.9548	-0.9548
		2			-1.4714	-1.4715
		3	-1.869075	-1.869074	-1.8691	-1.8691
		5	-2.500135	-2.500132		-2.5001
		10	-3.660379	-3.660372		-3.6604
	1	1				-0.8611
1	0				-0.5315	-0.5312
	1					-0.4505

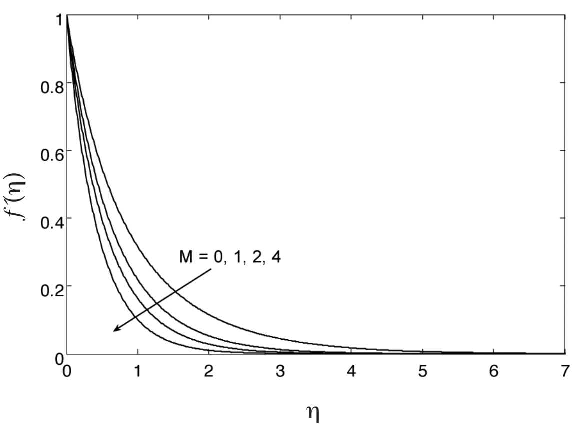

FIGURE 2. Velocity profiles for different values of M

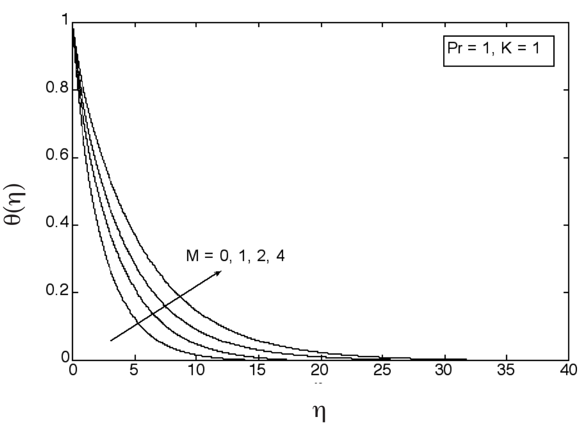

FIGURE 3. Temperature profiles for different values of M when Pr = 1 and K = 1

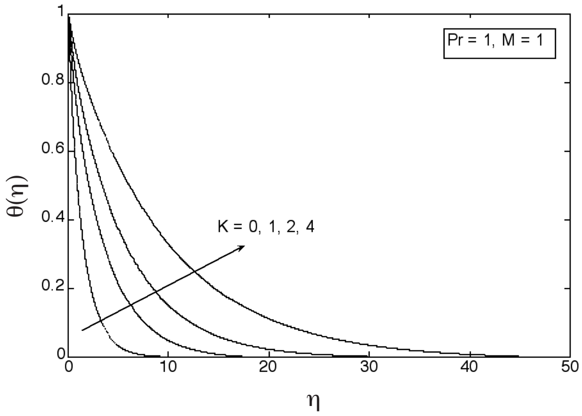

FIGURE 4. Temperature profiles for different values of *K* when Pr = 1 and *M* = 1

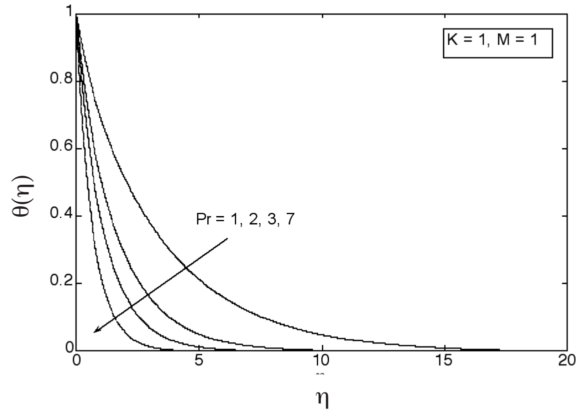

FIGURE 5. Temperature profiles for different values of Pr when *K* = 1 and *M* = 1

CONCLUSIONS

The effect of radiation on steady MHD boundary layer flow over an exponentially stretching sheet was investigated. The numerical results obtained agreed very well with previously reported cases available in the literature. It was found that the surface shear stress increases with the magnetic parameter *M*, while the heat transfer rate increases with Prandtl number Pr, but decreases with both magnetic parameter *M* and radiation parameter *K*.

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School of Mathematical Sciences
Faculty of Science and Technology
Universiti Kebangsaan Malaysia
43600 UKM Bangi, Selangor D.E.
Malaysia

*Corresponding author, email: anuar_mi@ukm.my

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